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RIGHT REVERSE DERIVATIONS ON PRIME RINGS

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ABSTRACT

In this paper some results concerning to right reverse derivations on prime rings with char $\neq 2$ are presented. If R be a prime ring with a non zero right reverse derivation d and U be the left ideal of R then R is commutative.

KEYWORDS: Prime Ring, Derivation, Reverse Derivation

INTRODUCTION

Bresar and Vukman [1] have introduced the notion of a reverse derivation. The reverse derivations on semi prime rings have been studied by Samman and Alyamani [2].

PRELIMINARIES

Through out, R will represent a prime ring with char $\neq 2$. We write $[x, y]$ for $xy - yx$. Recall that a ring R is called prime if $aRb=0$ implies $a=0$ or $b=0$. An additive mapping d from R into itself is called a derivation if $d(xy)=d(x)y+xd(y)$ for all $x, y\in R$ and is called a reverse derivation if $d(xy)=d(y)x+yd(x)$ for all $x, y\in R$.

MAIN RESULTS

Theorem 1

Let R be a prime ring with char $\neq$ 2, U a non-zero left ideal of R and d be a right reverse derivation of R. If U is non-commutative such that [x, d(x)] = 0 for all $x \in U$, then d = 0.

Proof

By linearizing the equation [d(x), x] = 0 which gives

[y, x]d(x) = 0, for all x, y $\in$ U (1)

We replace y by zy in equ.(1) and using (1), we get,

$\Rightarrow$ [zy, x]d(x) = 0

$\Rightarrow$ (z[y, x] + [z, x]y)d(x) = 0

$\Rightarrow$ z[y, x]d(x) + [z, x]yd(x) = 0

$\Rightarrow$ [z, x]y d(x) = 0, for all x, y, z $\in$ U (2)

By writing y by yr, $r\in R$ in equation (2), we obtain,

$\Rightarrow$ [z, x]yr d(x) = 0, for all x, y, $z\in U$ and $r\in R$.

If we interchange r and y, then we get,

$\Rightarrow$ [z, x]ry d(x) = 0,for all x, y, $z\in U$ and $r\in R$.

By primeness property, either [z, x] = 0 (or) d(x) = 0.

Since U is non-commutative, then d = 0.

Theorem 2

Let R be a prime ring with char $\neq 2$, U a left ideal of R and d be a non-zero right reverse derivation of R. If [d(y), d(x)] = [y, x] for all x, $y\in U$, then [x, d(x)] = 0 and hence R is commutative.

Proof

Given that [d(y), d(x)] = [y, x], for all x, $y\in U$

By taking yx instead of y in the hypothesis, then we get,

$\Rightarrow$ [yx, x] = [d(yx), d(x)]

$\Rightarrow$ y[x, x] + [y, x]x = [(d(x)y + d(y)x), d(x)]

$\Rightarrow$ [y, x]x = (d(x)y + d(y)x)d(x) – d(x)(d(x)y + d(y)x)

$\Rightarrow$ [y, x]x = d(x)yd(x) + d(y)x d(x) – d(x)d(x)y – d(x)d(y)x

Adding and subtracting d(y)d(x)x

$\Rightarrow$ [y, x]x = d(x)yd(x) + d(y)x d(x) – d(x)d(x)y – d(x)d(y)x + d(y)d(x)x – d(y)d(x)x

$\Rightarrow$ [y, x]x = d(x)yd(x) – d(x)d(x)y + d(y)x d(x) – d(y)d(x)x + d(y)d(x)x – d(x)d(y)x

$\Rightarrow$ [y, x]x = d(x) [yd(x) – d(x)y] + d(y)[x d(x) – d(x)x] + [d(y)d(x) – d(x)d(y)]x

$\Rightarrow$ [y, x]x = d(x)[y, d(x)] + d(y)[x, d(x)] + [d(y),d(x)]x

$\Rightarrow$ [y, x]x = d(x)[y, d(x)] + d(y)[x, d(x)] + [y, x]x

$\Rightarrow$ [y, x]x - [y, x]x = d(x)[y, d(x)] + d(y)[x, d(x)]

$\Rightarrow$ d(x)[y, d(x)] + d(y)[x, d(x)] = 0, for all x, $y\in U$ (3)

We replace y by cy = yc, where $c\in Z$ and using equation (3), we get,

$\Rightarrow$ d(x)[cy, d(x)] + d(cy)[x, d(x)] = 0

$\Rightarrow$ d(x) (c[y, d(x)] + [c, d(x)]y) + (d(y)c + d(c)y)[x, d(x)] = 0

$\Rightarrow$ d(x)c[y, d(x)] + d(x)[c, d(x)]y + d(y)c[x, d(x)] + d(c)y[x, d(x)] = 0

$\Rightarrow$ c d(x)[y, d(x)] + d(x)[c, d(x)]y + c d(y)[x, d(x)] + d(c)y[x, d(x)] = 0

$\Rightarrow$ -c d(y)[x, d(x)] + d(x)[c, d(x)]y + c d(y)[x, d(x)] + d(c)y[x, d(x)] = 0

$\Rightarrow$ d(x)[c, d(x)]y + d(c)y[x, d(x)] = 0

$\Rightarrow$ d(c)y[x, d(x)] = 0, for all x, y$\in$ U

Since 0 $\neq$ d(c) $\in$ Z and U is a left ideal of R, then we have, [x, d(x)] = 0, for all x$\in$ U.

By using the similar procedure as in Theorem: 1, then, we get, either [z, x] = 0 (or) d(x) = 0.

Since d is non-zero, then [z, x] = 0.

Hence R is commutative.

Theorem 3

Let R be a prime ring with char$\neq$2, U a left ideal of R and d be a non-zero right reverse derivation of R. If [d(y), d(x)] = 0, for all x, y$\in$ U, then R is commutative.

Proof

Given that [d(y), d(x)] = 0, for all x, y$\in$ U

By taking yx instead of y in the hypothesis, then we get,

$\Rightarrow$ [d(yx), d(x)] = 0

$\Rightarrow$ [(d(x)y + d(y)x), d(x)] = 0

$\Rightarrow$ [d(x)y, d(x)] + [d(y)x, d(x)] = 0

$\Rightarrow$ d(x)[y, d(x)] + [d(x),d(x)]y + d(y)[x, d(x)] + [d(y),d(x)]x = 0

$\Rightarrow$ d(x)[y, d(x)] + d(y)[x, d(x)] = 0, for all x, y$\in$ U (4)

The proof is now completed by using equation (3) of Theorem: 2.

Hence R is commutative.

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